Lecture 3

Homework

(60) How much will you have if you deposit $200 dollars at the end of the year for the next 20 years if the account earns 4% interest per year. (Ans: $5955.615715)

Ans:

$$\left(\frac{(1+i)^n-1}{i}\right)D = \left(\frac{(1.04)^{20}-1}{.04}\right)200 = 5955.62$$

(23) George is borrowing $20,000 and will pay 8.5% interest. He will pay off the loan in three annual installments, $6,000 at the end of the first year, $7,000 at the end of the second year, and a final payment at the end of the third year. What should the amount of the final payment be?

George owes $20,000 + interest =

$$20000(1.085)^3 = 25{,}545.78$$

George paid

$$6000\underbrace{(1.085)^2}_{\text{interest saved}} + 7000\underbrace{(1.085)}_{\text{interest saved}} = 14{,}658.35$$

George still owes

$$25{,}545.78 - 14{,}658.35 = 10{,}887.43$$

(28) Amount you will have if you invest \$75 at the end of each month for 10 years if the account pays 7.5% compounded monthly.

Every month you earn $\frac{.075}{12}$ interest.

You make $10 \times 12 = 120$ deposites

$$\text{Ans} = \left(\frac{\left(1+\frac{.075}{12}\right)^{120}-1}{\frac{.075}{12}} \right) \times 75$$

$$= 13,344.78$$

(44) What is the present value of \$25,000 ten years from now at 8% interest?

$$25,000(1.08)^{-10} = 11,579.84$$

(46) What is the future value of \$25,000 ten years from now at 8% interest?

$$25000(1.08)^{10} = 53,973.12$$

(51) An insurance company earns 7% on their investments. How much must they have on reserve (the present value of claims) on January 1, 2002 to cover the claims for the next 3 years, if they expect claims of \$500,000 for 2002, \$300,000 for 2003 and \$250,000 for 2004. For sake of simplicity, assume that the claims are all paid on January 1 of the following year. Thus the 2002 claims are paid on January 1 of 2003, the 2003 claims are paid on January 1 of 2004, etc.

We want the PV of the claims

$PV = 500000(1.07)^{-1} + 300000(1.07)^{-2}$

$+ 250000(1.07)^{-3}$

$= 933,395.81$

(53) An interest rate of 6% compounded three times a year is equivalent to what rate of interest compounded twice a year.

$\left(1+\frac{.06}{3}\right)^3 = \left(1+\frac{i}{2}\right)^2$

$\sqrt{1.061208} = 1+\frac{i}{2}$

$i = 2\left(\sqrt{1.061208} - 1\right) = 0.\cancel{60299}$ $.060299$

Comment on roundoff

Lecture

Example 14. I plan to retire at age 70, at which time I will withdraw \$5,000 per month for 20 years from my IRA. Assuming that my funds are invested at 4.7% interest, compounded monthly how much must I have accumulated in my IRA?

Solution

we earn $\frac{.047}{12} = i$ per month.

Let P = amount I nee at age 70.

Over 20 years P grows to

$$\left(1+\frac{.047}{12}\right)^{240} \cdot P = 2.555 \cdot P$$

On the othe hand our withdrawls reduce the accumulation by

$$\left(\frac{\left(1+\frac{.047}{12}\right)^{240}-1}{\frac{.047}{12}}\right) 5000 = 1{,}985{,}471.11$$

Hence $$P = \frac{1985\ 471.11}{2.555} = 777{,}005.53$$

$$P = \underbrace{\left(1+\frac{.07}{12}\right)^{-240}}_{PV} \cdot \underbrace{\left(\frac{\left(1.\frac{04}{12}\right)^{240}-1}{\frac{.047}{12}}\right) \cdot 5000}_{\text{accumulated value}}$$

Remark This example illustrates a general principle:

The amount of money in our account must equal the present value of all of the payments out of the account.

Example 15. I borrow \$25,000 to buy a car on which I pay \$1000 down and make monthly payments at the end of the month over the next 5 years. If I pay 7% interest, compounded monthly, what are my monthly payments?

Solution I borrow 24,000.

Let P be the monthly payment.

The bank insists that we eventually pay both the principle and interest.

Hence afte 5 years our payments must accumulate to a total of

$$24\,000\left(1+\frac{.07}{12}\right)^{12\cdot 5} = 34,023,01$$

Our payments accumulate to

$$\left(\frac{\left(1+\frac{.07}{2}\right)^{12\cdot 5}-1}{\frac{.07}{2}}\right)P = 79,5929\,P$$

Hence $P = \dfrac{34\,023.01}{79,5929} = 475,23$

<u>Other approach</u>

PV (payments) = amount borrowed

$$\left(1+\frac{.07}{12}\right)^{-12\cdot 5}\left(\frac{\left(1+\frac{.07}{12}\right)^{12\cdot 5}-1}{\frac{.07}{12}}\right)P = 24,000$$

EXAMPLE 15. I borrow \$25,000 to buy a car on which I pay \$1000 down and make monthly payments at the end of the month over the next 5 years. If I pay 7% interest, compounded monthly, ~~what are my monthly payments?~~ How much do I still owe at the end of the third year.

Solution I owe the principal plus 3 years of interest minus my accumulated payments. Thus I owe

?

$$24{,}000\left(1+\frac{.07}{12}\right)^{12\cdot 3} - \frac{\left(\left(1+\frac{.07}{12}\right)^{12\cdot 3}-1\right)}{\frac{.07}{12}} \cdot 475.23$$

$n-1$

$1+r+\cdots+r^{n-1} = \frac{1-r^{n+1}}{1-r}$

$1+r+r^2$

$$= 10{,}614.23$$

This is what I would need to pay if I paid off my loan early

50,000

imm -7,000

Gam 2,000

-15,000

40,000

$50000 = (1+i)\, 40000$

$i = ?$

$\cancel{(1+i)}$

$(1+i)^{\frac{11}{12}}$